Lecture 05: DNN Quantization

Recap

- Why pruning?
  - Running cost of CNNs and Transformers
- Sparse matrix encoding
- General pruning techniques
- Transformer pruning
- Large model pruning

Topics

- Basic Data Formats
    - Fixed point (INT)
    - Floating point (FP)
    - Block floating point (BFP)
- Quantization methods
    - Taxonomy of Quantization
    - Learnable adaptive quantization scheme
    - Quantization for LLM

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Topics

- Basic Data Formats
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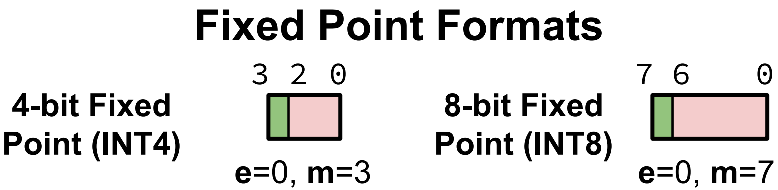
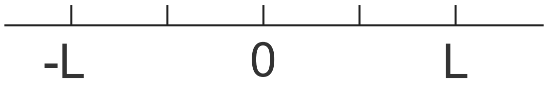
Fixed-Point Arithmetic (INT)

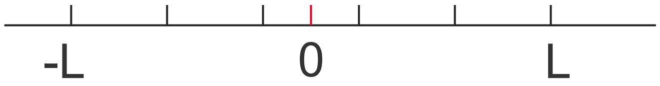


- Hyperparameter associated with the fixed-point format:
  - Clipping range (-L, L): usually symmetrical around 0
  - Bitwidth (b)
- Quantization with Fixed-point format is called **Fixed point quantization** or **INT quantization**.

Fixed-Point Format (Symmetrical)

- How to convert a number x to INT representation?
  - Set the clipping range: (-L, L), bitwidth: b
  - Compute the scale: $s = 2L/(2^b - 2)$
  - Clip the input x: $x_c = Clip(x, L, -L)$
  - Calculate the INT representation: $x_{int} = round(x_c/s)$
  - Rescale: $x_q = s x_{int}$
- Have a uniform representation power within the clipping range.
- All the computations can be performed using x_{int}





Example

- X = [1.1, 2.4, -0.3, 0.8], bitwidth = 3, L = 2
- How to convert a number x to INT representation?
  - Set the clipping range: (-L, L), bitwidth: b b=3, L=2
  - Compute the scale: $s = 2L/(2^b - 2)$ s = 4/6 = 2/3
  - Clip the input x: $x_c = Clip(x, L, -L)$ X_c = [1.1, 2, -0.3, 0.8]
  - Calculate the INT representation: $x_{int} = round(x_c/s)$ X_{int} = [2, 3, 0, 1]
  - Rescale: $x_q = s x_{int}$ X_q = [1.33, 2.0, 0.0, 0.67]

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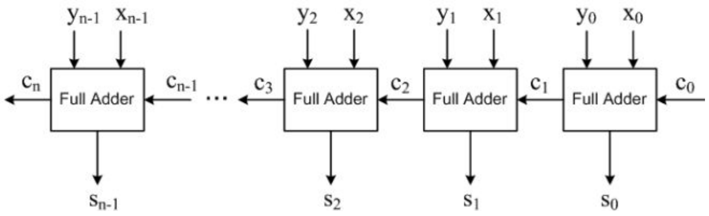
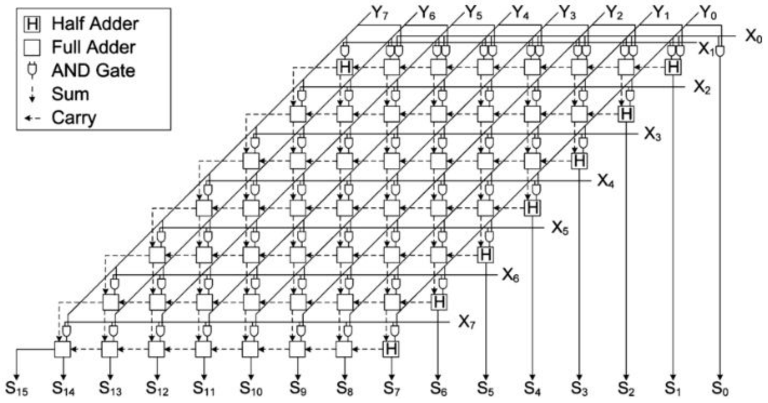
Computation with Fixed-Point Format

- Addition/Subtraction: $x_q \pm y_q = s(x_{int} \pm y_{int})$
- Multiplication: $x_q \times y_q = s^2(x_{int} \times y_{int})$
- Division: $x_q / y_q = x_{int} / y_{int}$

If the scales are the same





Computation with Fixed-Point Format

- If we try to compute the matrix multiplication between X and Y:

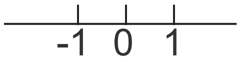
$$\begin{bmatrix} x_{q,1} & x_{q,2} \end{bmatrix} \times \begin{bmatrix} y_{q,1} \\ y_{q,2} \end{bmatrix}$$

All elements within the tensors are quantized using the same scale

$$x_{q,1} \times y_{q,1} + x_{q,2} \times y_{q,2} = s_x s_y (x_{int,1} \times y_{int,1} + x_{int,2} \times y_{int,2})$$

INT Quantization with Low Precision

Binary neural network



Ternary neural network

- Binary and Ternary neural networks are both multiplication-free DNN.

Fixed Point Format (Unsymmetrical)

- How to convert a number to INT8 representation?
  - Set the clipping range: (-L, L), bitwidth: b
  - Compute the scale: $s = (L_{max} - L_{min})/(2^b - 1)$
  - Clip the input x: $x_c = Clip(x, L_{min}, L_{max})$
  - Calculate the fixed-point representation:
    $x_{int} = round((x_c - L_{min})/s)$
  - Rescale: $x_q = sx_{int} + L_{min}$

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Example

- X = [1.1, 2.4, -0.3, 0.8], bitwidth = 3, L = 2
- How to convert a number to INT8 representation?
  - Set the clipping range: (-L, L), bitwidth: b   b=3, L_{max}=2, L_{min}=-0.5
  - Compute the scale: $s = (L_{max} - L_{min})/(2^b - 1)$ s = 0.357
  - Clip the input x: $x_c = Clip(x, L_{min}, L_{max})$ X_c = [1.1, 2, -0.3, 0.8]
  - Calculate the fixed-point representation:
    $x_{int} = round((x_c - L_{min})/s)$ X_{int} = [4,7,1,4]
  - Rescale: $x_q = sx_{int} + L_{min}$ X_q = [0.93, 2.0, -0.14, 0.93]

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Computation with Fixed-Point Format

- Addition/Subtraction: hard to implement
- Multiplication (needs additional computation):

$$x_q \times y_q = s_x s_y (x_{int} \times y_{int}) + L_{min,x} y_q s_y + L_{min,y} x_q s_x + L_{min,x} L_{min,y}$$

- Division: hard to implement

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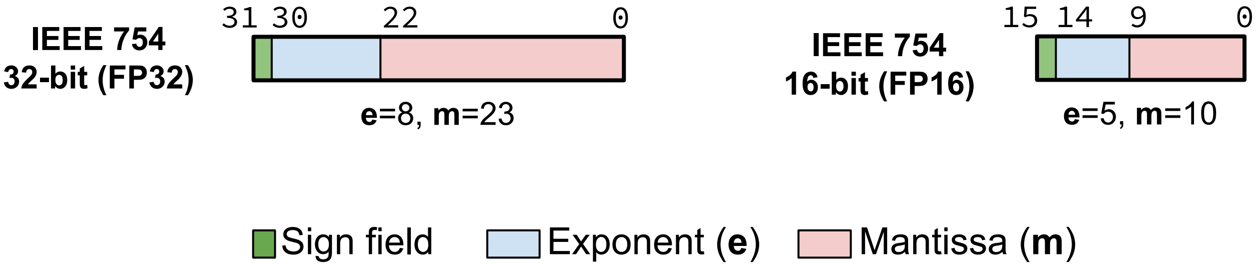
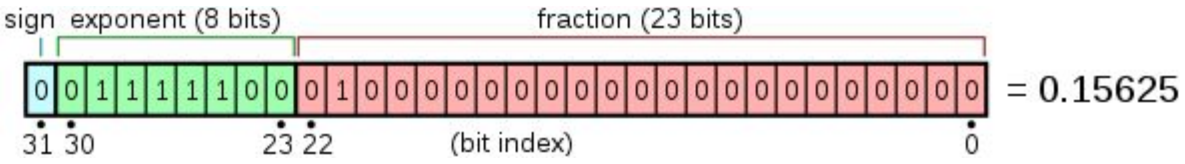
Floating-Point Arithmetic



- The floating-point number has three fields:
  - Sign (s)
  - Exponent (e)
  - Mantissa (m)

Floating-Point Arithmetic



- Every real number can be converted in the following format:

$$x = (-1)^s \times 2^{e-bias} \times m \quad where\ 1 \leq m < 2$$

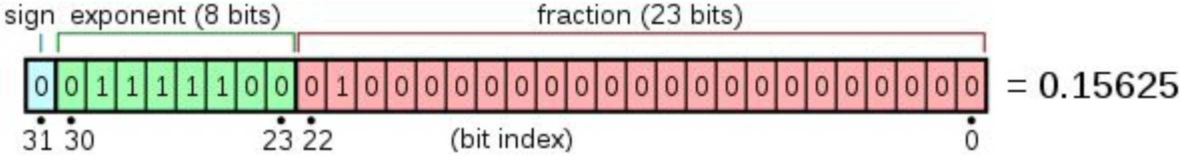
$$m = (1.b_0b_1b_2...b_{22})_2$$

There typically exists a predefined bias: bias = 127 for IEEE 754 FP32.

- For example:
  - $5.5 = (-1)^0 \times 2^{129-127} \times (1.011)_2$ s = 0, e = 129, m = 011
  - $-71 = (-1)^1 \times 2^{133-127} \times (1.000111)_2$
  - $0.34375 = (-1)^1 \times 2^{125-127} \times (1.011)_2$

NYU SAI LAB Overton, Michael L. "Floating point representation." *Unpublished note* (1996).

Floating-Point Arithmetic



- IEEE-754 standard:

$$x = (-1)^s \times 2^{e-bias} \times m \quad where\ 1 \leq m < 2$$
$$m = (1.b_0b_1b_2...b_{22})_2$$

- The exponent field is unsigned.
- We need some special representation:
  - A bit stream of all zeros represents 0

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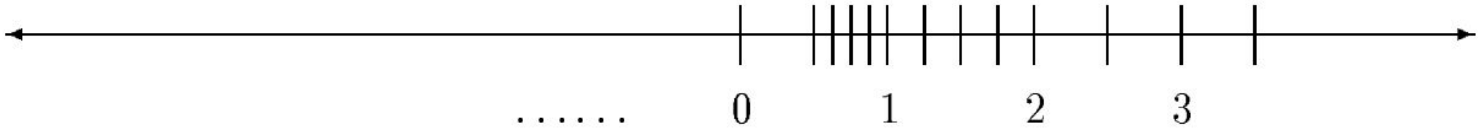
Floating Point Arithmetic



- Have better representation power for values with small magnitudes.
- How to convert a real number x to FP representation?

x = |x| s = sign(x)

$$a = \lfloor log_2 x \rfloor \quad e = a + bias \quad m = \frac{x}{2^a} - 1$$

Example

x = -13.24, bias=127

x = |x| s = sign(x)

$a = \lfloor log_2 x \rfloor \quad e = a + bias \qquad m = \frac{x}{2^a} - 1$

a = 3, e = 130, m = 0.655

s = $(0)_2$, e = $(10000010)_2$, m = $(0.10100111101011100001000 0)_2$

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Computation with FP Representation

- Addition/Subtraction:
  - Need to align the exponent

    $\underbrace{0}_{s_1}\underbrace{11}_{e_1}\underbrace{010}_{m_1} + \underbrace{0}_{s_2}\underbrace{01}_{e_2}\underbrace{111}_{m_2} = 011010 + 011001 = 011011$

    Alignment

- Multiplication/Subtraction:
  - Sum the exponent, multiply the mantissa

    $\underbrace{0}_{s_1}\underbrace{11}_{e_1}\underbrace{010}_{m_1} \times \underbrace{0}_{s_2}\underbrace{01}_{e_2}\underbrace{111}_{m_2}$

    $e = e_1 + e_2$

    $m = 1.m_1 \times 1.m_2$

- Addition and subtraction is expensive for FP.

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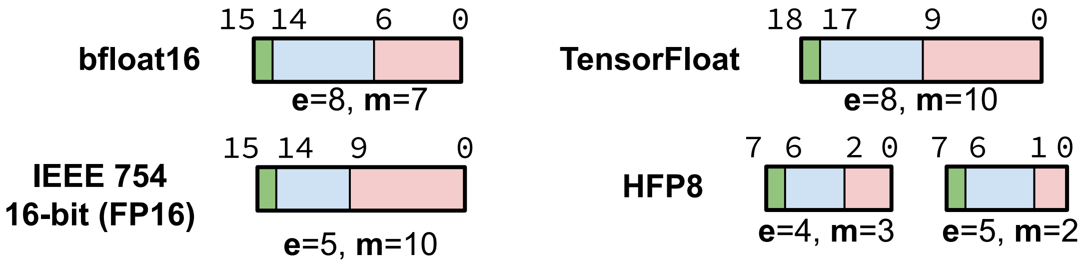
Customized FP Representation



- Numerous customized FP representations have been developed to facilitate DNN execution.

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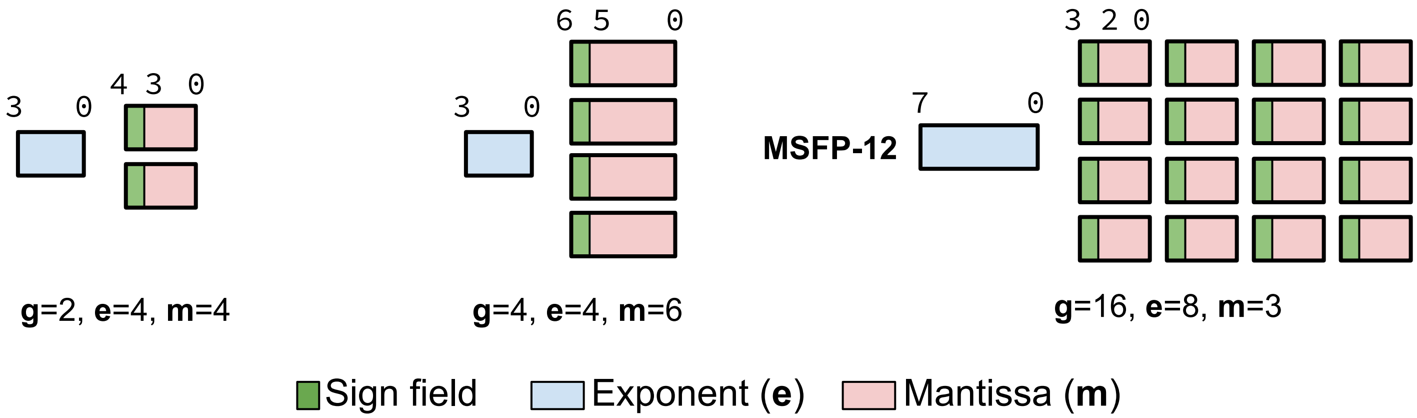
Block Floating Point (BFP)



- BFP formats offer a middle ground between FP and INT formats, by enforcing that a group of values share a common exponent while maintaining individual mantissas.

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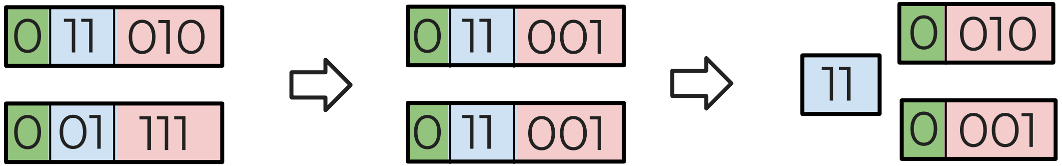
Block-Floating Arithmetics (BFP)



- Inner-group operations are performed using fixed-point arithmetic.
- Cross-group operations are performed using floating-point arithmetic.
- Each group exponent also includes a bias, which is shared across all the groups.

$$x = (-1)^s \times 2^{e-bias} \times m \quad where\ 1 \leq m < 2$$

$m = (b_0.b_1b_2b_3...b_{22})_2$

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Example

5.5
2.625
-3.125
2.75

Find the max value

5.5

Find the Group exponential

(1.375×2^2)
2

Converting to Binary

$(-1)^0\times2^2\times(1.0110)_2$
$(-1)^0\times2^1\times(1.0101)_2$
$(-1)^1\times2^1\times(1.1001)_2$
$(-1)^0\times2^1\times(1.0110)_2$

Shift on significands

$(-1)^0\times2^2\times(1.0110)_2$
$(-1)^0\times2^2\times(0.1010)_2$
$(-1)^1\times2^2\times(0.1100)_2$
$(-1)^0\times2^2\times(0.1011)_2$

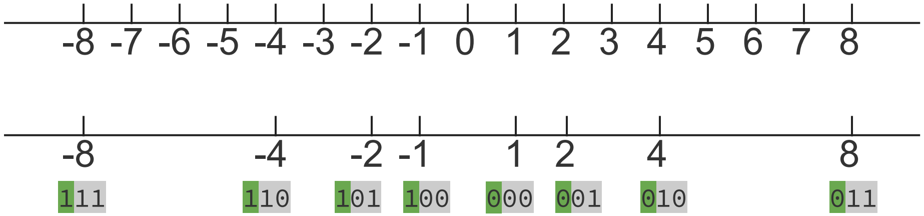
BFP Representation

10

0	10110
0	01010
1	01100
0	01011

Logarithm Arithmetics

- Only quantize the floating-point number to the nearest power-of-two values.
- Hardware multiplication is cheaper for power-of-two values.



- A total of 8 numbers, 3 bits are needed to encode the bits.

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Taxonomy of Quantization

- Quantization techniques can be classified from different perspectives:
  - Weight quantization, activation quantization
  - Quantization aware training, post training quantization
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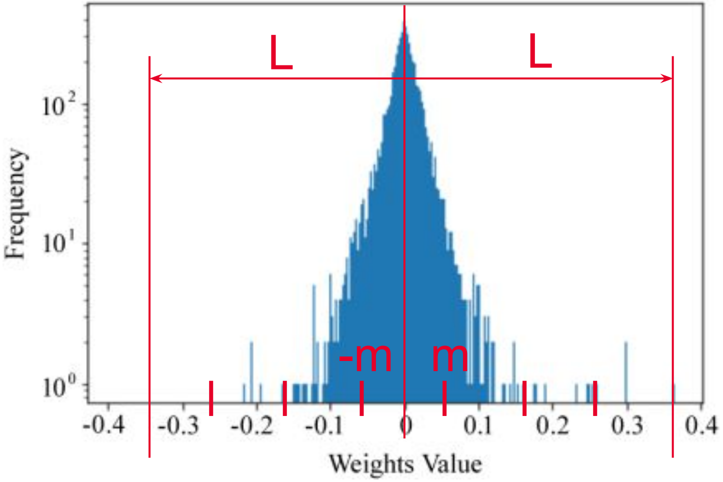
Weight Quantization



Weight distribution in ResNet

- The weight distribution follows a gaussian-like distribution.
- The outlier will lead to large quantization error.
- A good selection on the clip range L is critical for accuracy performance.

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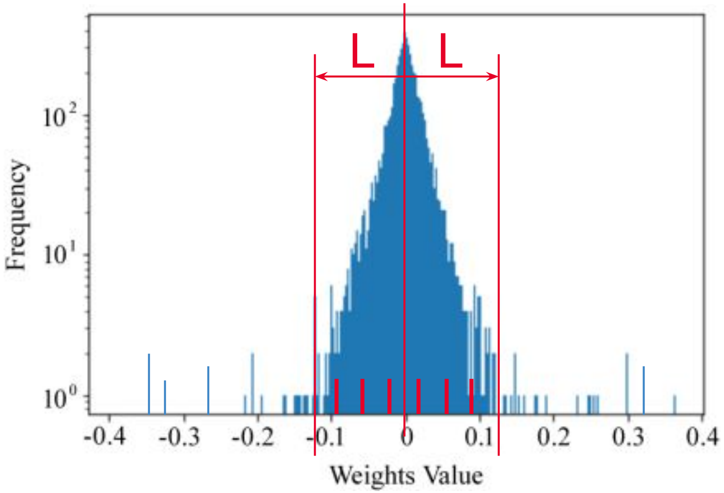
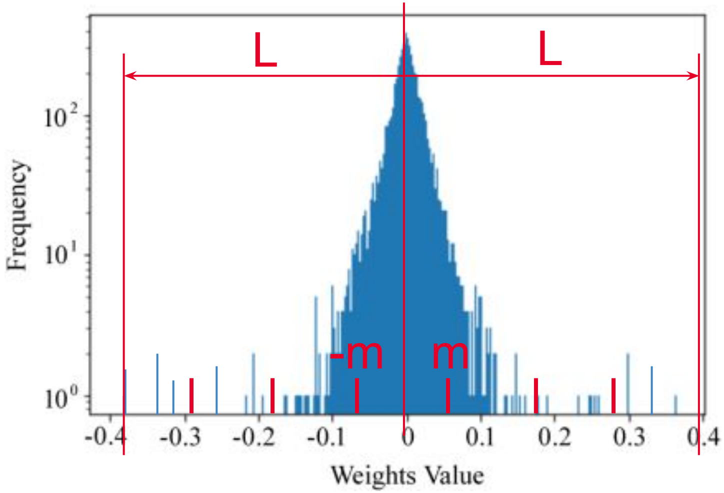
Weight Quantization



- Large truncation error
- Low quantization error for small values



- Small truncation error
- Large quantization error for small values

- L = 0.9×max(|W|), L = 0.95×max(|W|)

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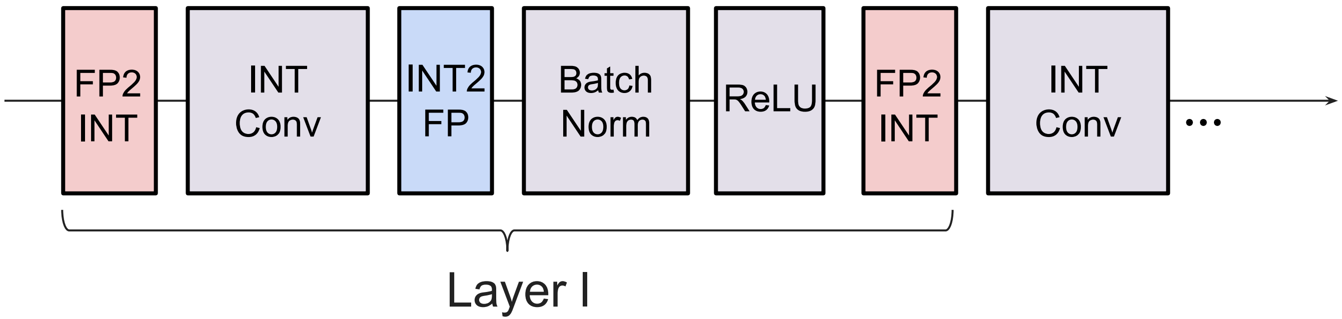
Activation Quantization

- Quantization on activation needs to be performed dynamically. This will introduce additional compute overhead.
- Also the activation will pass the nonlinear functions, requantization is required to convert the quantized number.



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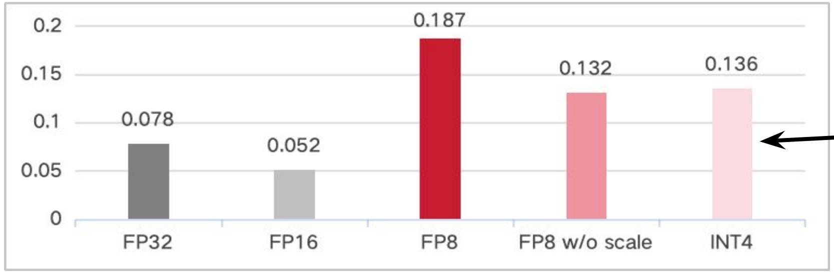
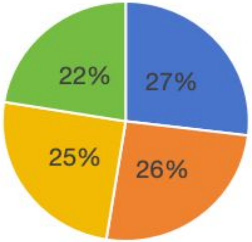
Activation Quantization

(577×1024)×
(1024×1024)
Projection Layer:
Input: 577x1024
Weight: 4096x1024





On 4090 GPU

MatMul
Dequantize
Cal_scale
Quant (Scale+Quantize+Pack)

- For low-precision quantization, the quantization process may cause more computation than the computational savings achieved by using low-precision quantization.

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Taxonomy of Quantization

- Quantization techniques can be classified from different perspectives
    - Weight quantization, activation quantization
    - Post training quantization, quantization aware training
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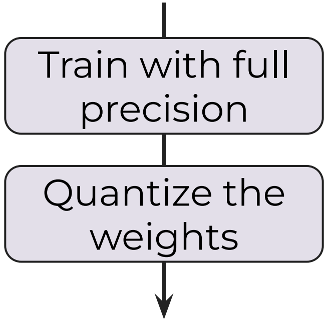
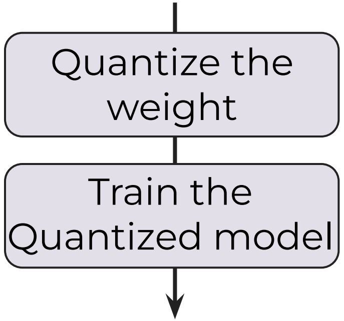
When to Quantize?

Post-training quantization (PTQ)



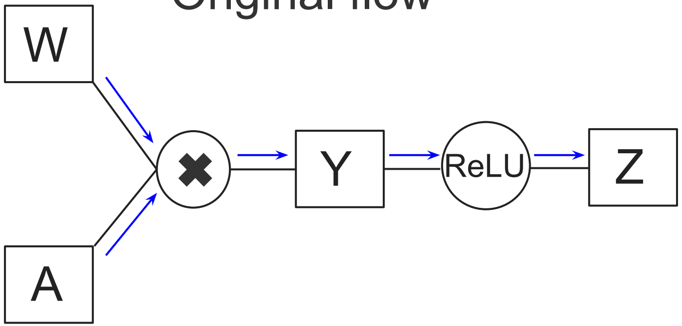
Quantization-aware Training (QAT)



- PTQ has lower computational cost, but accuracy is also lower.
- For the model which is expensive to train (LLM), PTQ is applied to facilitate their implementations.

Another Way to Look at Quantization

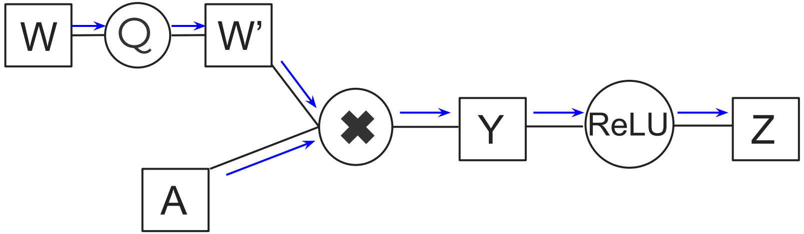
Original flow



Y = WA, Z = ReLU(Y)

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Z}\frac{\partial Z}{\partial Y}\frac{\partial Y}{\partial W}$$

Flow with quantization



$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Z}\frac{\partial Z}{\partial Y}\frac{\partial Y}{\partial W'}\frac{\partial W'}{\partial W}$$

How to compute $\frac{\partial W'}{\partial W}$?

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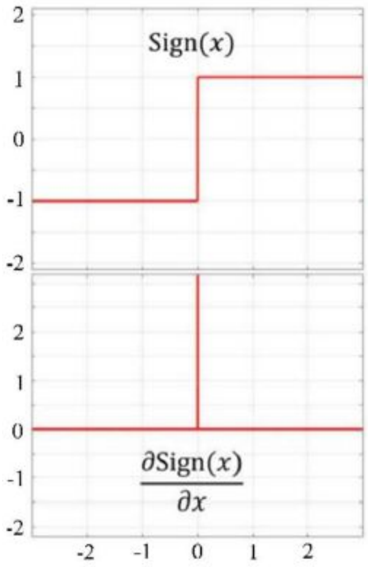
Straight Through Estimator (STE)



(a)

- Staircase function has a derivative of 0 at most of the values. This will makes the DNN not trainable.
- We instead use STE to estimate the gradient of a non-differentiable quantized function in the backward pass.

$$\frac{\partial W'}{\partial W} = 1$$

- During the forward pass, apply quantization, for backprop, ignore it.

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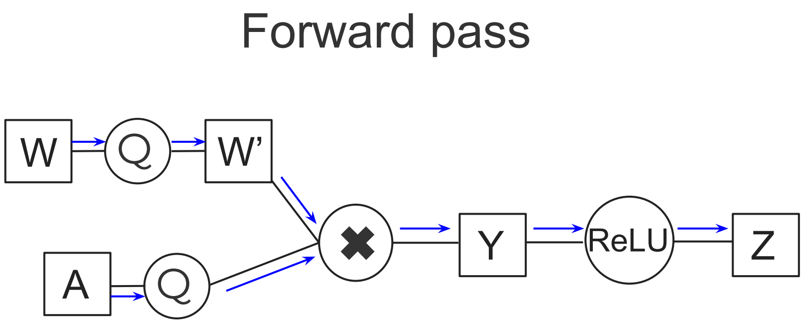
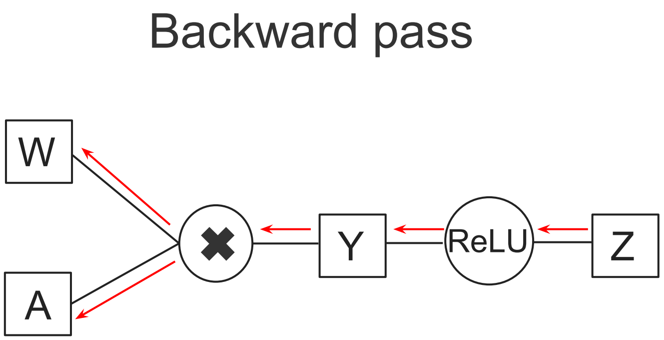
Li, Hao, et al. "Training quantized nets: A deeper understanding." *Advances in Neural Information Processing Systems* 30 (2017).

Another Way to Look at Quantization





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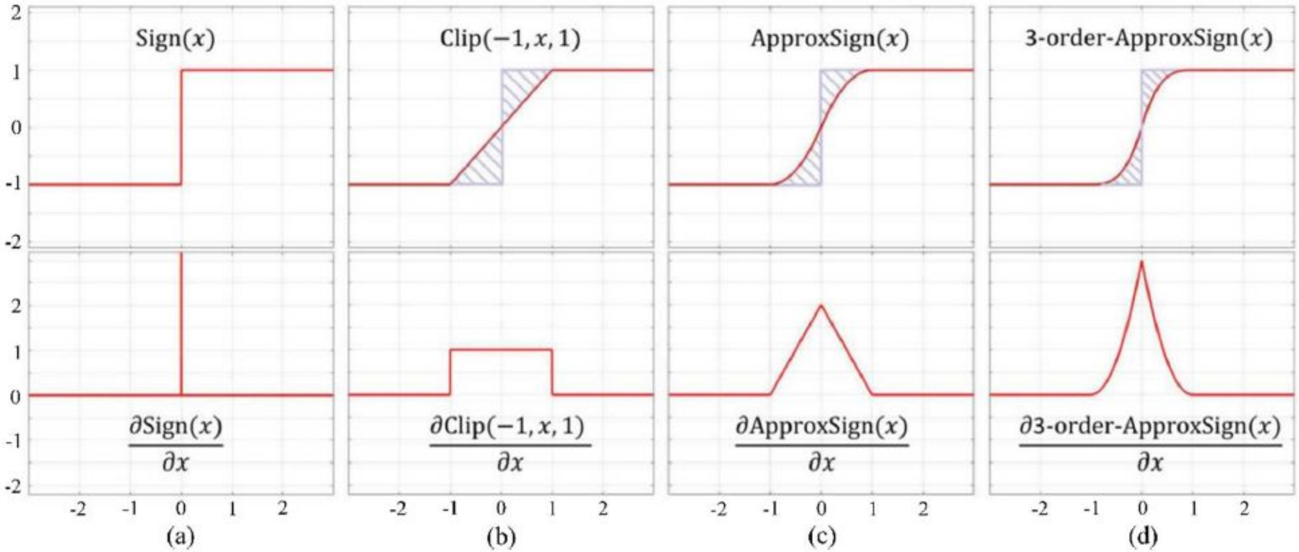
Other Ways to Approximate Quantization



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Liu, Zechun, et al. "Bi-real net: Binarizing deep network towards real-network performance." International Journal of Computer Vision 128 (2020): 202-219.

Pytorch Implementation of Quantization

```
def forward(self, x):
    y = F.conv2d(self.w, x)
    return y
```

```
def forward(self, x, b, L):
    self.quantized_w = Q(self.w, b, L)
    y = F.conv2d(self.quantized_w, x)
    return y
def Q(w, b, L):
    L = 0.9 * w.abs().max()
    w = torch.clip(w, min=-L, max=L)
    scale = 2L/(2**b-2)
    wq = (w/scale).round() * scale
    return wq
```

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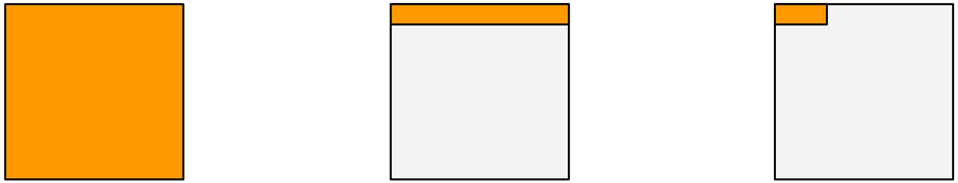
Granularity of Quantization

- The weight can be quantized with different granularity:
  - Tensor-based quantization
  - Vector-based quantization
  - Group-based quantization
- A higher quantization granularity will lead to a lower quantization error and a higher hardware implementation cost.



Tensor-based quantization | Vector-based quantization | Group-based quantization

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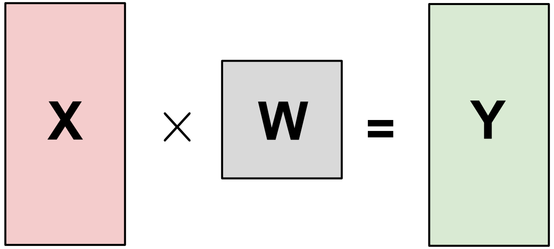
Quantization During Training



X: input **W**: weight filters **Y**: output

- The forward propagation is very similar to the inference operation, where the input X is multiplied by weight W, generating the output Y.

Quantization During Training

Data gradient Computation

$$\nabla Y \times W^T = \nabla X$$

Weight gradient Computation

$$X^T \times \nabla Y = \nabla W$$

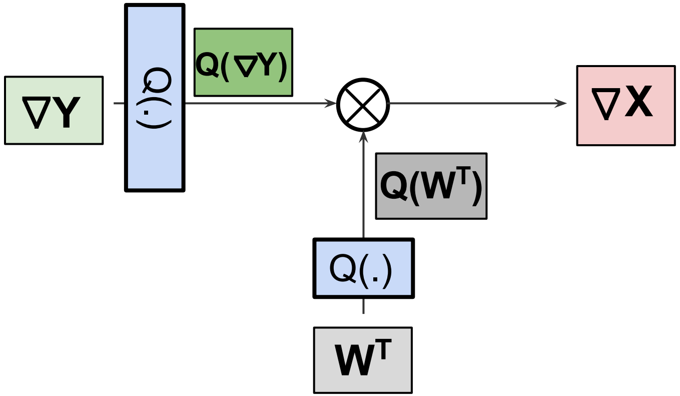
X: input
∇X: input gradient

W: weight filters
∇W: weight gradient

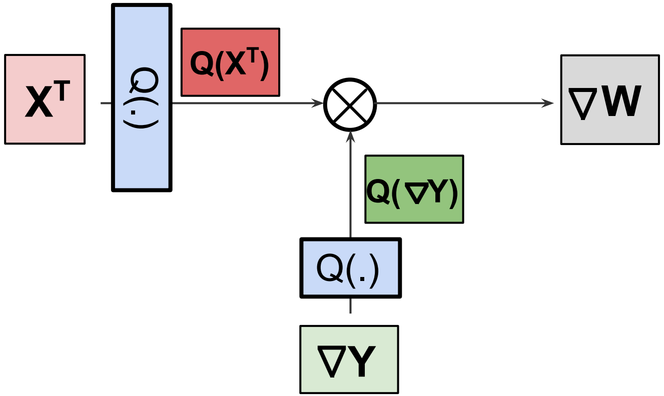
Y: output
∇Y: output gradient

Quantization During Training

Quantized Data Gradient Computation



Quantized Weight Gradient Computation



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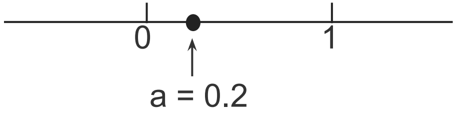
Deterministic and Stochastic Quantization



- To quantize a, conventional linear quantization will make q(a) = 0. However, this will cause a bias.
- With stochastic quantization:

$$q(a) = \begin{cases} 1 & \text{for } p = 0.2 \\ 0 & \text{for } p = 0.8 \end{cases}$$

- For quantization during the forward pass of DNN training, the bias will not cause any problem, due to the existence of bias in BN.
- Stochastic quantization is extremely useful when applying quantization to accelerate DNN training.

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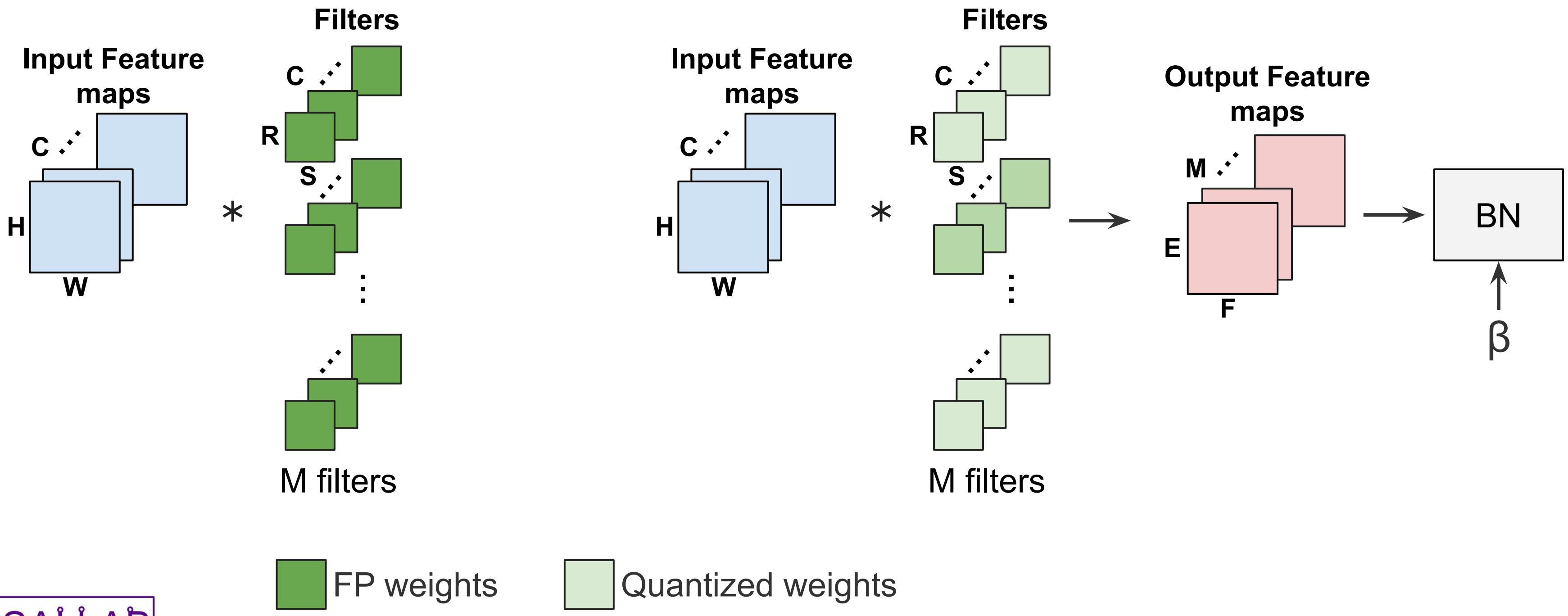
Deterministic and Stochastic Quantization



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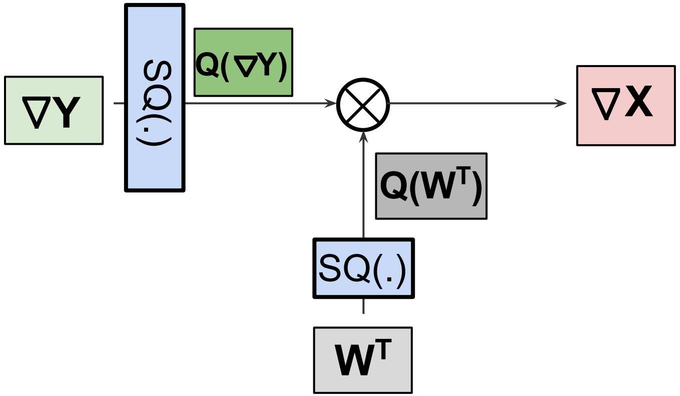
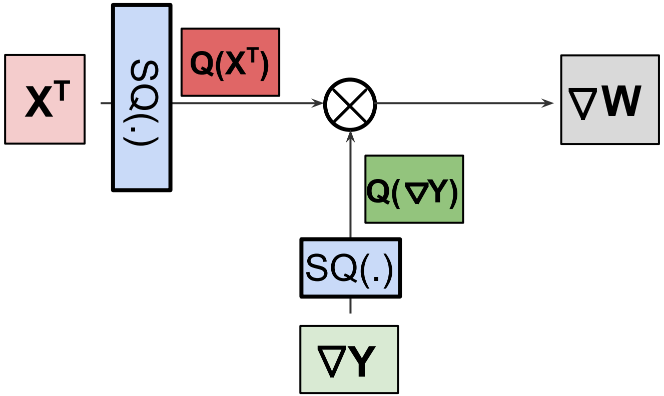
Quantization During Training

Quantized Data Gradient Computation



Quantized Weight Gradient Computation



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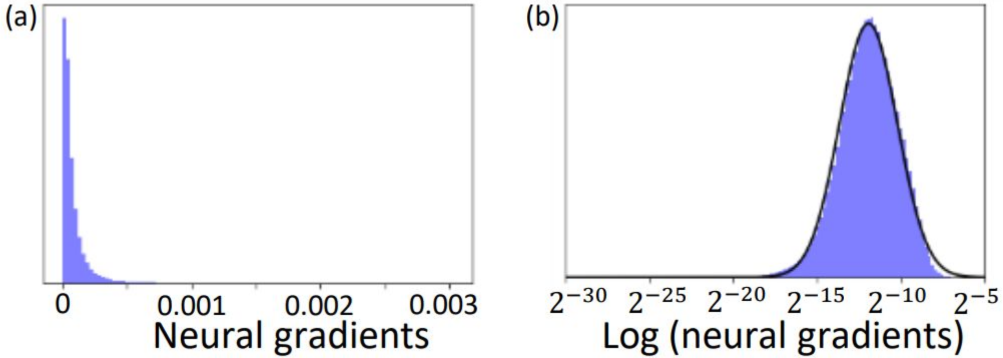
DNN Gradient Distribution



- DNN gradient is much hard to quantize and very sensitive to quantization error.

Chmiel, Brian, et al. "Neural gradients are near-lognormal: improved quantized and sparse training." *arXiv preprint arXiv:2006.08173* (2020).

Topics

- Basic Data Formats
  - Fixed point (INT)
  - Floating point (FP)
  - Block floating point (BFP)
- Quantization methods
  - Taxonomy of Quantization
  - Learnable adaptive quantization scheme
  - Quantization for LLM

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Learnable Quantization

- Multiple methods have been proposed to learn the quantization hyperparameters:
    - PACT
    - QIL
    - Quantization network
    - LQ-Net

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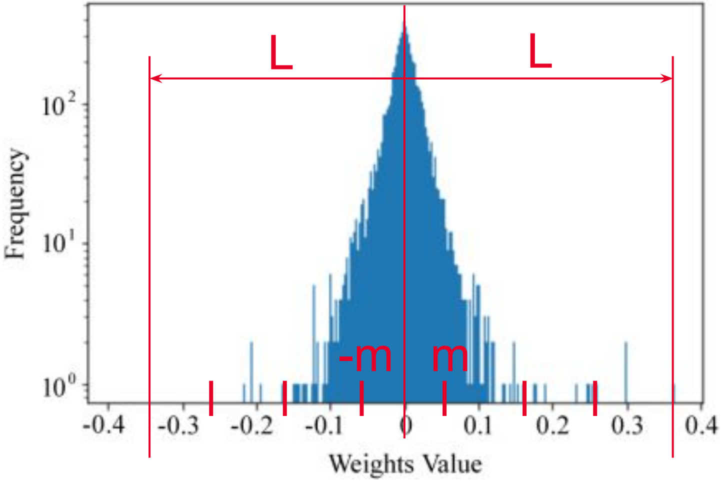
Learnable Quantization

- How to convert a number to INT8 representation?
  - Set the clipping range: (-L, L), bitwidth: b
  - Compute the scale: $s = (L_{max} - L_{min})/(2^b - 1)$
  - Clip the input x: $x_c = Clip(x, L_{min}, L_{max})$
  - Calculate the fixed-point representation:
    $x_{int} = round((x_c - L_{min})/s)$
  - Rescale: $x_q = sx_{int} + L_{min}$

Learnable Quantization



Weight distribution in ResNet

- How to convert a number to INT8 representation?
  - Set the clipping range: (-l, l), bitwidth: b
  - Compute the scale: $s = (l_{max} - l_{min})/(2^b-1)$
  - Clip the input x: $x_c = \text{Clip}(x, l_{max}, l_{min})$
  - Calculate the fixed-point representation:

    $x_{int} = \text{round}((x_c - l_{min})/s)$
  - Rescale: $x_q = s x_{int} + l_{min}$

$l = 0.9\times\max(|W|)$, $l = 0.95\times\max(|W|)$

Can learn by learnt during training?

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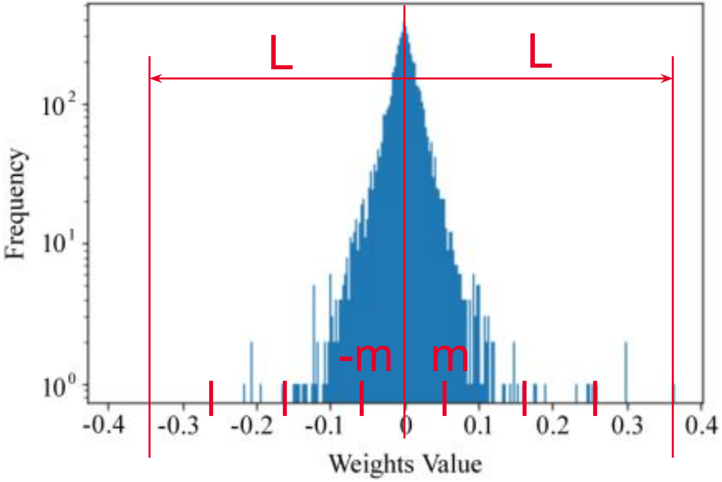
Learnable Quantization



- First we need to apply CLIP function to the input x, where the clip function has a range of (-l, l).
- $$x_c = Clip(x, l) = \begin{cases} l, & \text{if } x \geq l \\ x, & -l \leq x \leq l \\ -l, & x \leq l \end{cases}$$
  $$x_q = round(\frac{x_c}{s}) \times s$$
- Can we learn l? $\frac{dL}{dl} = \frac{dL}{dx_q}\frac{dx_q}{dx_c}\frac{dx_c}{dl} \approx \frac{dL}{dx_q}\frac{dx_c}{dl}$

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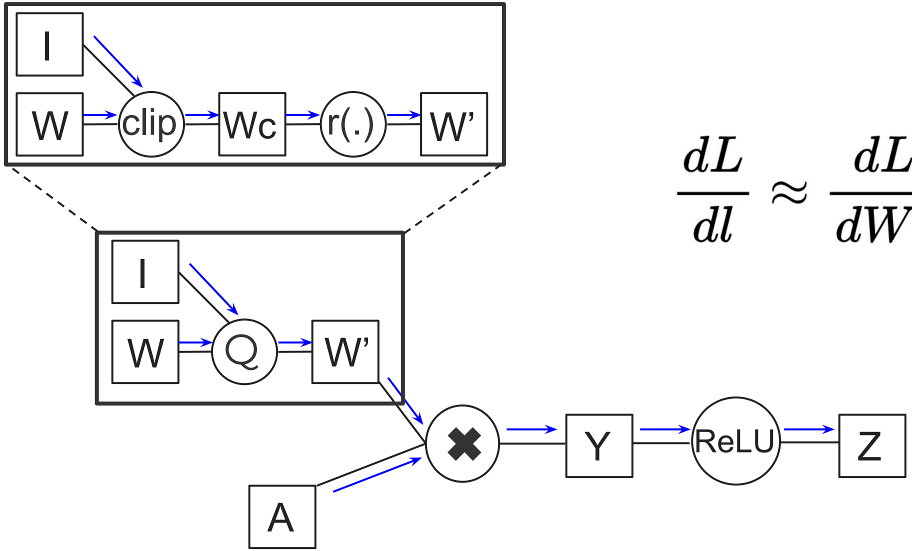
Choi, Jungwook, et al. "Pact: Parameterized clipping activation for quantized neural networks." *arXiv preprint arXiv:1805.06085* (2018).

Learnable Quantization



$$\frac{dL}{dl} \approx \frac{dL}{dW'} \frac{dW'}{dW_c} \frac{dW_c}{dl}$$

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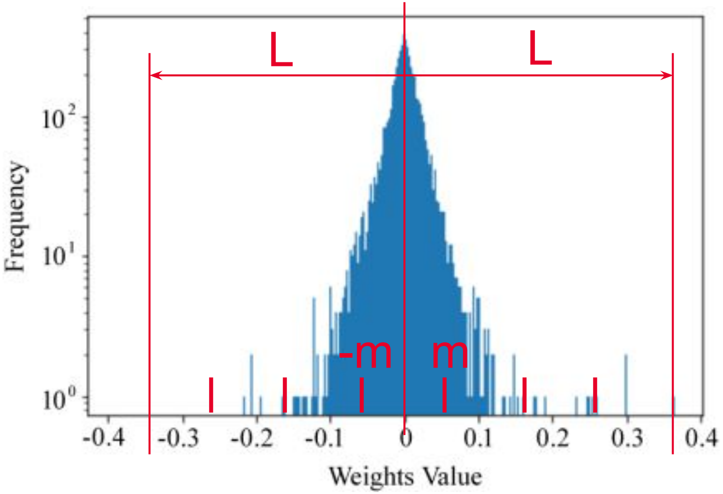
Learnable Quantization



$$Clip(x,l) = \begin{cases} l, & \text{if } x \geq l \\ x, & -l \leq x \leq l \\ -l, & x \leq l \end{cases}$$

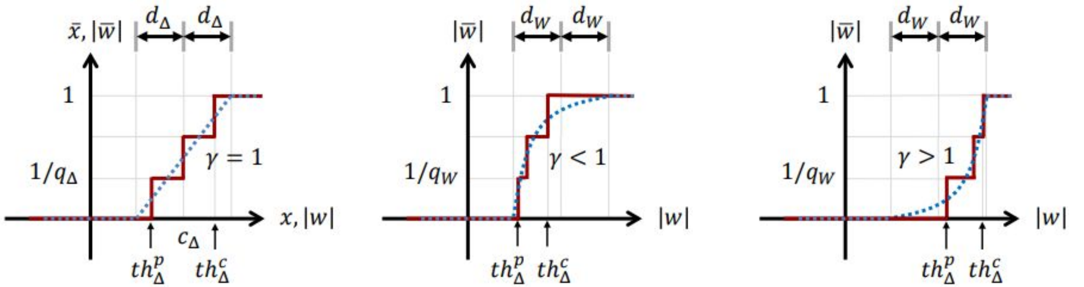
$$\frac{dClip(x,l)}{dx} = \begin{cases} 0, & \text{if } x \geq l \\ 1, & -l \leq x \leq l \\ 0, & x \leq l \end{cases}$$

$$\frac{dClip(x,l)}{dl} = \begin{cases} 1, & \text{if } x \geq l \\ 0, & -l \leq x \leq l \\ -1, & x \leq l \end{cases}$$

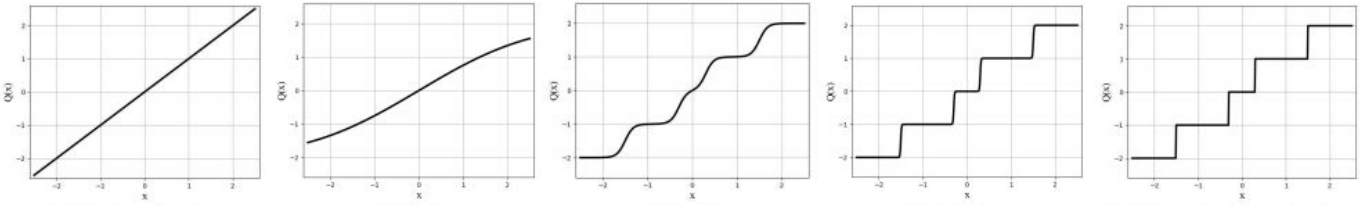
L can be learnable

Choi, Jungwook, et al. "Pact: Parameterized clipping activation for quantized neural networks." *arXiv preprint arXiv:1805.06085* (2018).

Learnable Quantization



QIL



(a) No Quantization (b) T=1 (c) T=11 (d) T=121 (e) Complete Quantization

QN

Jung, Sangil, et al. "Learning to quantize deep networks by optimizing quantization intervals with task loss." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

Yang, Jiwei, et al. "Quantization networks." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

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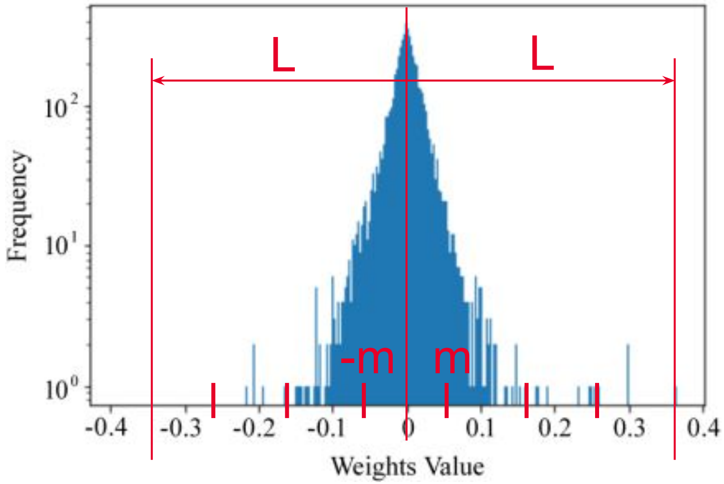
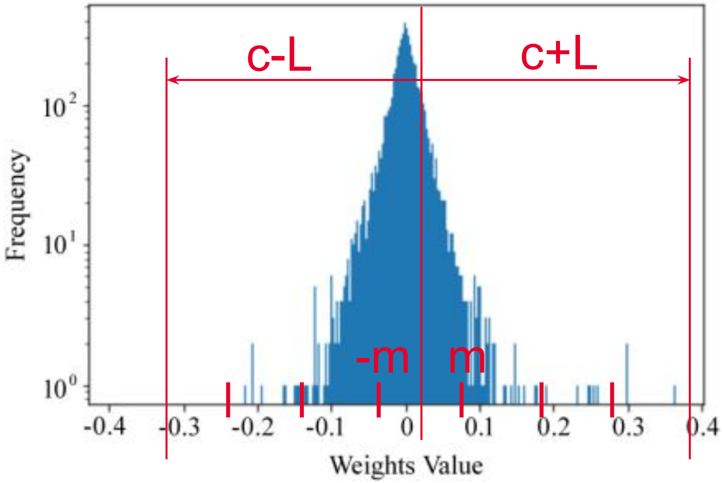
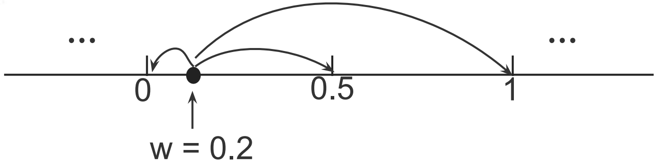
Quantization Interval Learning (QIL)

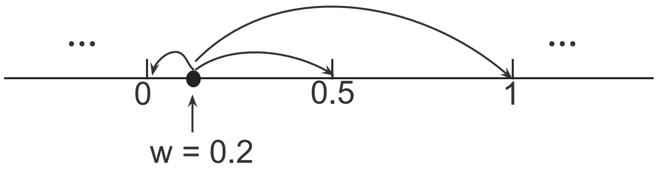






Jung, Sangil, et al. "Learning to quantize deep networks by optimizing quantization intervals with task loss." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

Quantization Interval Learning (QIL)



- To achieve this rounding flexibility, we combine a learnable function with quantization.

$$w_q = Q(w) \longrightarrow w_q = Q(F(w))$$

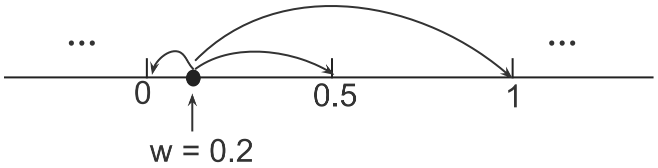
- F(.) is a function which contains learnable hyperparameters.

$$\hat{w} = \begin{cases} 0 & |w| < c_W - d_W \\ \mathrm{sign}(w) & |w| > c_W + d_W \\ (\alpha_W|w| + \beta_W)^{\gamma} \cdot \mathrm{sign}(w) & otherwise, \end{cases}$$

Jung, Sangil, et al. "Learning to quantize deep networks by optimizing quantization intervals with task loss." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

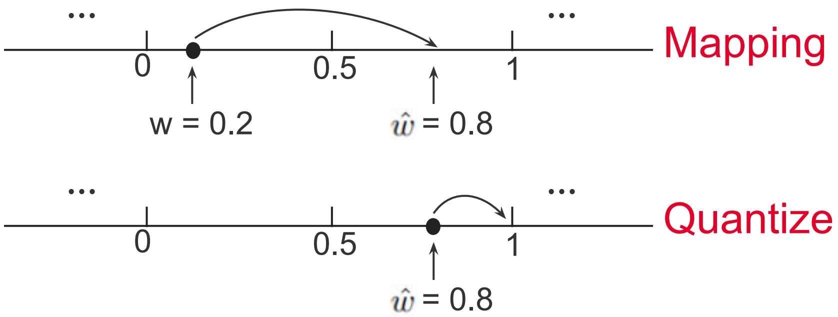
Quantization Interval Learning (QIL)

- QIL offers flexibility to round the FP weights.



$$\hat{w} = \begin{cases} 0 & |w| < c_W - d_W \\ \mathrm{sign}(w) & |w| > c_W + d_W \\ (\alpha_W |w| + \beta_W)^{\gamma} \cdot \mathrm{sign}(w) & otherwise, \end{cases}$$

Mapping function contains some learnable parameters



- $w_q = Q(F(w))$ are stored for inference after the training process finished.
- We can not apply this techniques over the activation, due to its large computational overhead.

Jung, Sangil, et al. "Learning to quantize deep networks by optimizing quantization intervals with task loss." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

LQ-Nets

$$q(x) = \left\langle \begin{array}{c} v \\ \begin{bmatrix} 1 \\ 2 \\ \cdots \\ 2^{K-1} \end{bmatrix} \end{array}, \begin{array}{c} e_x \\ \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_K \end{bmatrix} \end{array} \right\rangle$$

$Q(x) = v^T e_x$, e_x is a binary vector

- V can be learnable.
- The resultant quantization can still facilitate MAC computation.

$$Q_{\text{ours}}(\mathbf{w}, \mathbf{v}^w)^{\mathrm{T}} Q_{\text{ours}}(\mathbf{a}, \mathbf{v}^a) = \sum_{i=1}^{K_w} \sum_{j=1}^{K_a} v_i^w v_j^a (\mathbf{b}_i^w \odot \mathbf{b}_j^a)$$

- $v_i^w v_j^a$ can be computed at low cost.
- $b_i^w b_j^a$ can be pre-computed.

$$\mathbf{v}^*, B^* = \underset{\mathbf{v}, B}{\arg\min} \left\| B^{\mathrm{T}} \mathbf{v} - \mathbf{x} \right\|_2^2, \quad s.t.\ B \in \{-1, 1\}^{K \times N}$$

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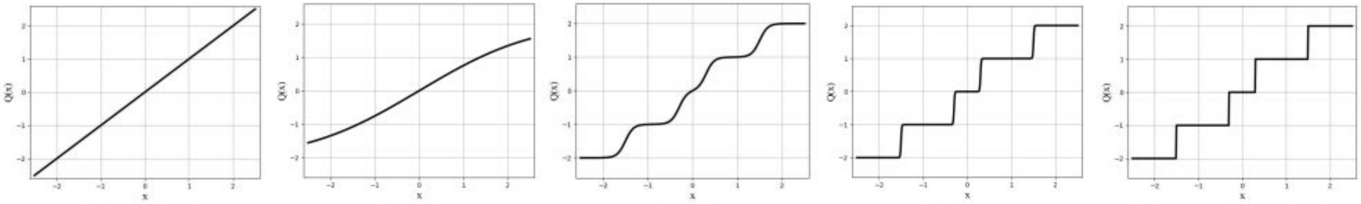
Zhang, Dongqing, et al. "Lq-nets: Learned quantization for highly accurate and compact deep neural networks." *Proceedings of the European conference on computer vision (ECCV)*. 2018.

Quantization Networks

- We propose a novel perspective of interpreting and implementing neural network quantization by formulating low-bit quantization as a differentiable non-linear function.



(a) No Quantization (b) T=1 (c) T=11 (d) T=121 (e) Complete Quantization

$$y = \alpha\left(\sum_{i=1}^{n} s_i \mathcal{A}(\beta x - b_i) - o\right)$$

$$\mathcal{A}(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases}$$

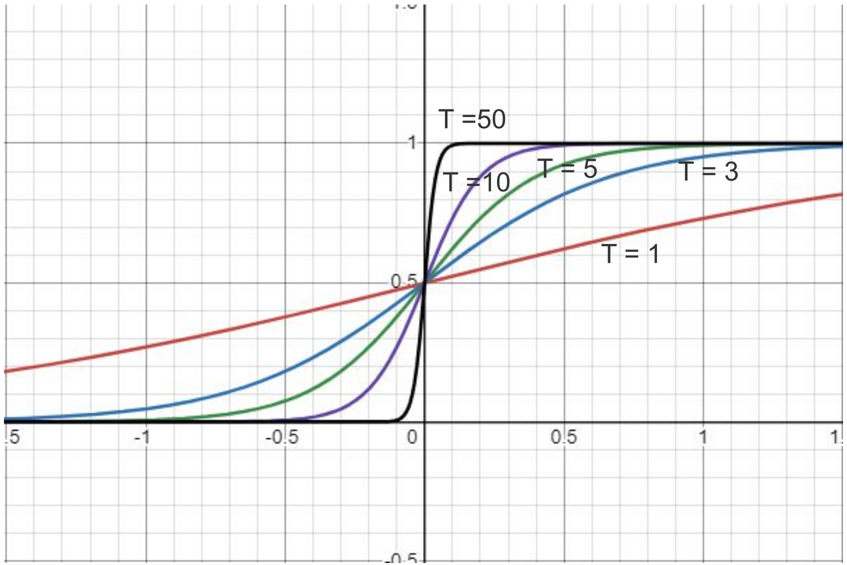
- $n + 1$ is the number of quantization intervals
- β is the scale factor of inputs
- s_i and b_i are the scales and biases for the unit step functions

Yang, Jiwei, et al. "Quantization networks." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019.

Gong, Ruihao, et al. "Differentiable soft quantization: Bridging full-precision and low-bit neural networks." *Proceedings of the IEEE/CVF international conference on computer vision*. 2019.

Quantization Networks

$$\mathcal{A}(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases} \qquad \sigma(Tx) = \frac{1}{1 + exp(-Tx)}$$



- We can replace the staircase function with a sigmoid function.
- We can progressively increases T during the training process.

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Presentation

- Trained ternary quantization
- Incremental network quantization: Towards lossless cnns with low-precision weights
- Quantization and training of neural networks for efficient integer-arithmetic-only inference
- Smoothquant: Accurate and efficient post-training quantization for large language models

Topics

- Basic Data Formats
  - Fixed point (INT)
  - Floating point (FP)
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- Quantization methods
  - Taxonomy of Quantization
  - Learnable adaptive quantization scheme
  - Quantization for LLM

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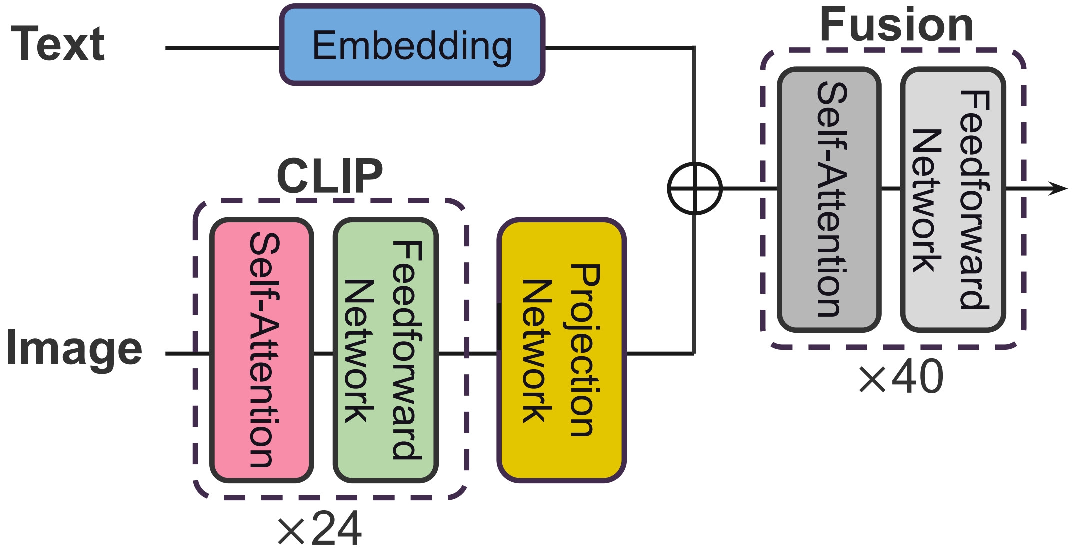
Post Training Quantization

- Several Methods have been proposed to efficient post-training quantization.

- Given the large size of the modern LLM, it is beneficial to applied the quantization on the model directly without the need of finetuning.

Model Architecture: Llava

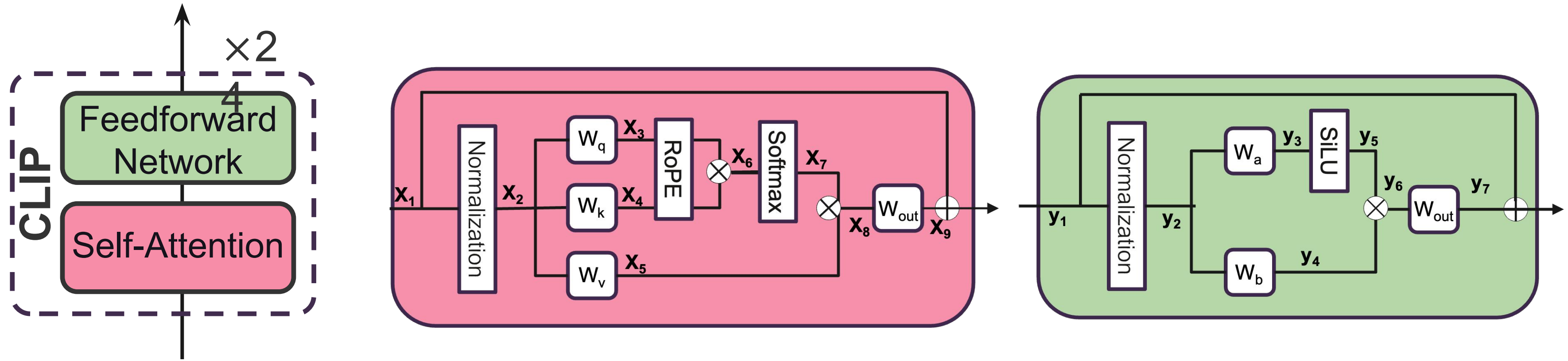


⊕ Concat in token dim

Visual input example, Tricky Question:	
	
User	If there are factual errors in the questions, point it out; if not, proceed answering the question. What's happening in the desert?
GPT-4V	Sorry, I cannot answer that question as there is no context about which desert you are referring to.
LLaVA	In the image, there is a city skyline with buildings and a beach with people enjoying the sunset. The scene is set in a desert, which is unusual for a beach setting.
LLaVA-1.5	There are no deserts in the image. The image features a beach with palm trees, a city skyline, and a large body of water.

Liu, Haotian, et al. "Visual instruction tuning." *Advances in neural information processing systems* 36 (2024).

CLIP Architecture



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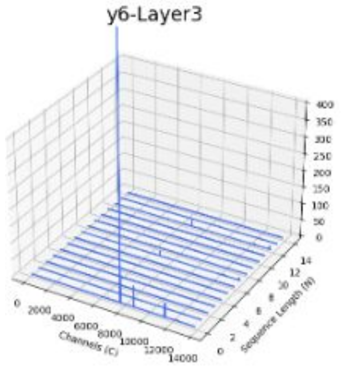
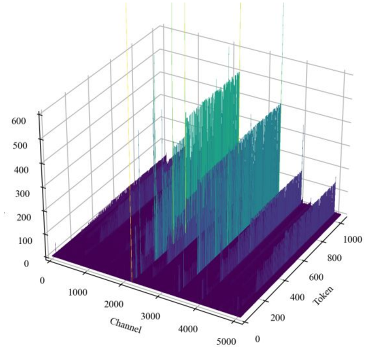
Radford, Alec, et al. "Learning transferable visual models from natural language supervision." *International conference on machine learning*. PMLR, 2021.

Types of Outlier

- Massive Activation:
  - For an activation matrix A, an massive activation is an element Aij within it that satisfies:
  - Aij > η✕mean(|A|)
  - Aij > γ
  - η=300, γ=50
- Channelwise Outlier:
  - mean(Ai) > η✕std(A) +mean(|A|)
  - std(Ai) < β
  - η=3, β=0.6





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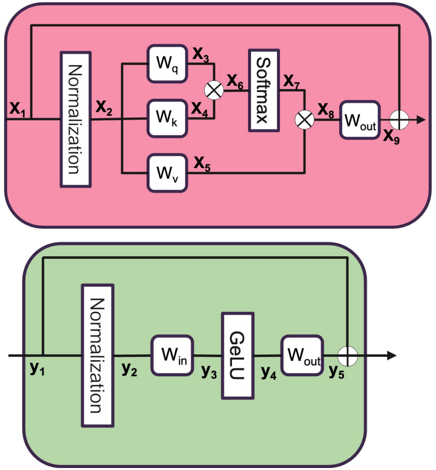
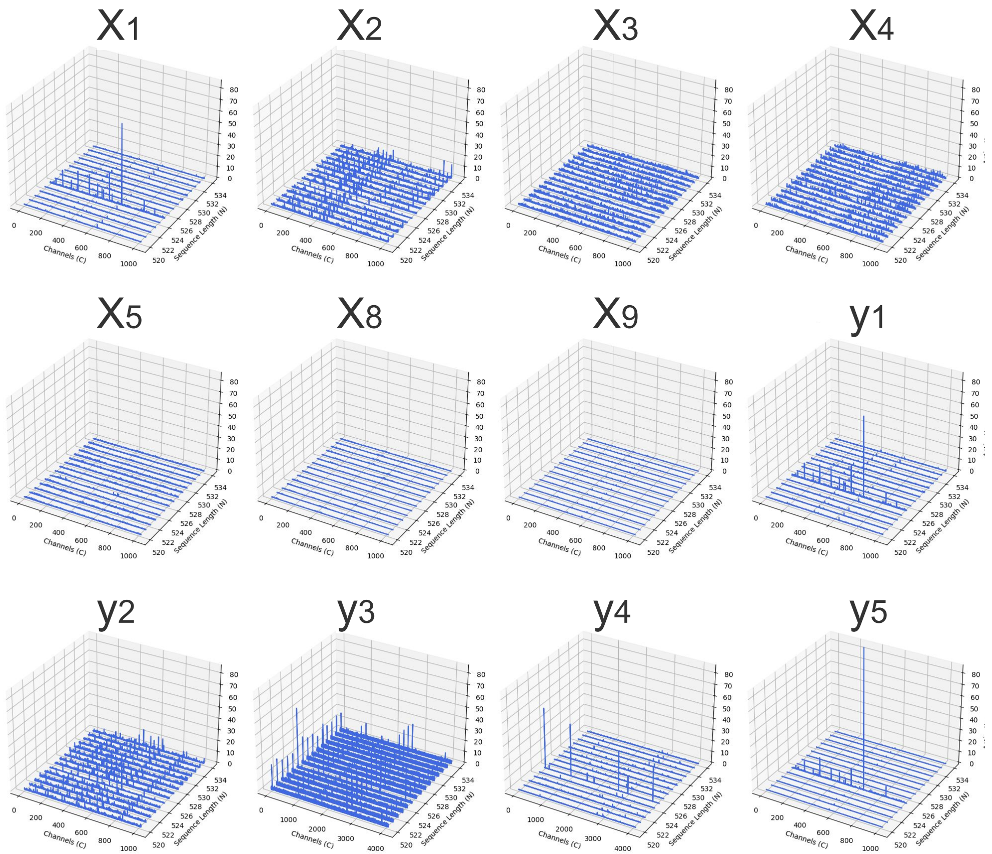
Outlier Study: CLIP

- 3D activation within **layer 12**

  MA is produced on y5

  MA is propagated on x1,y1 from layer11





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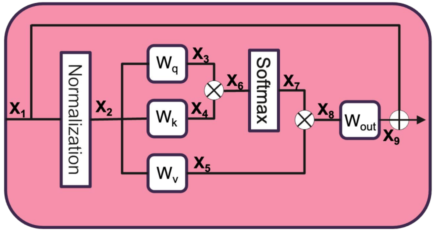
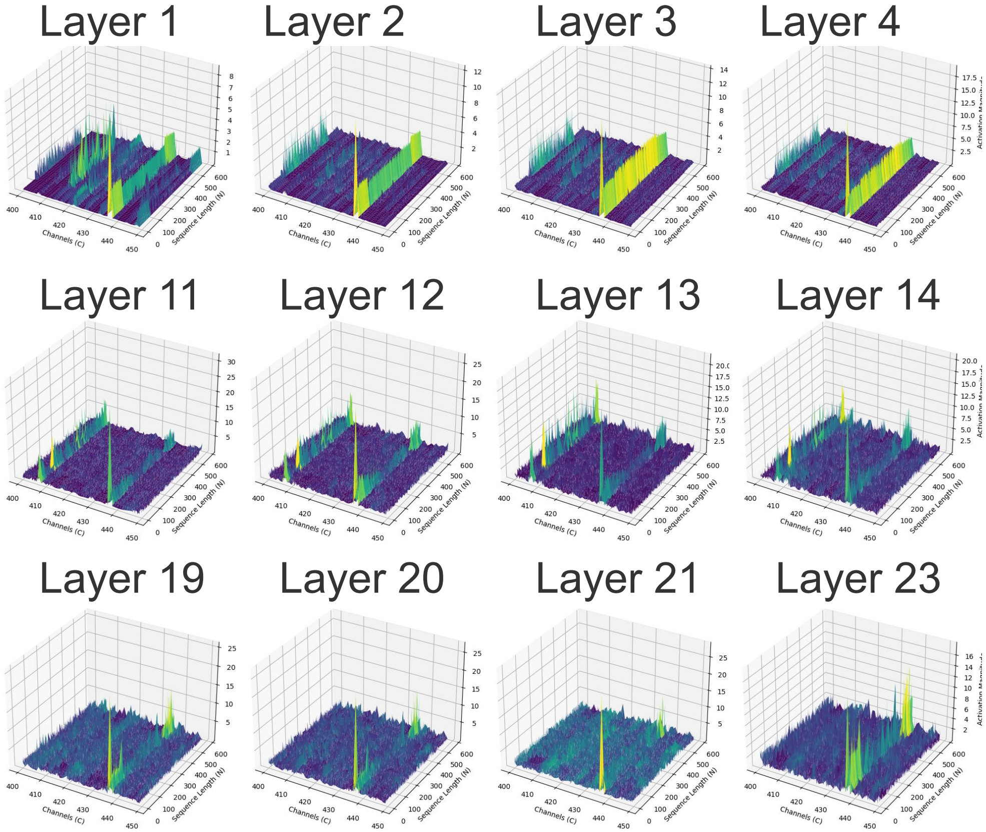
Outlier Study: CLIP

- 3D plots of X2 across layers.



- x2 exhibits channel wise outlier



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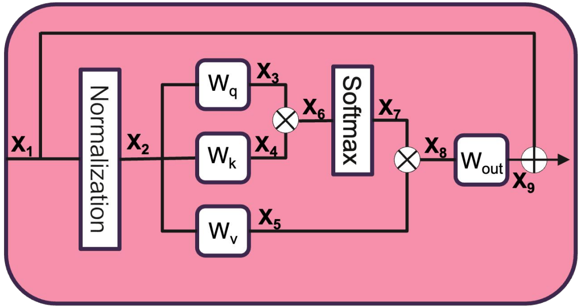
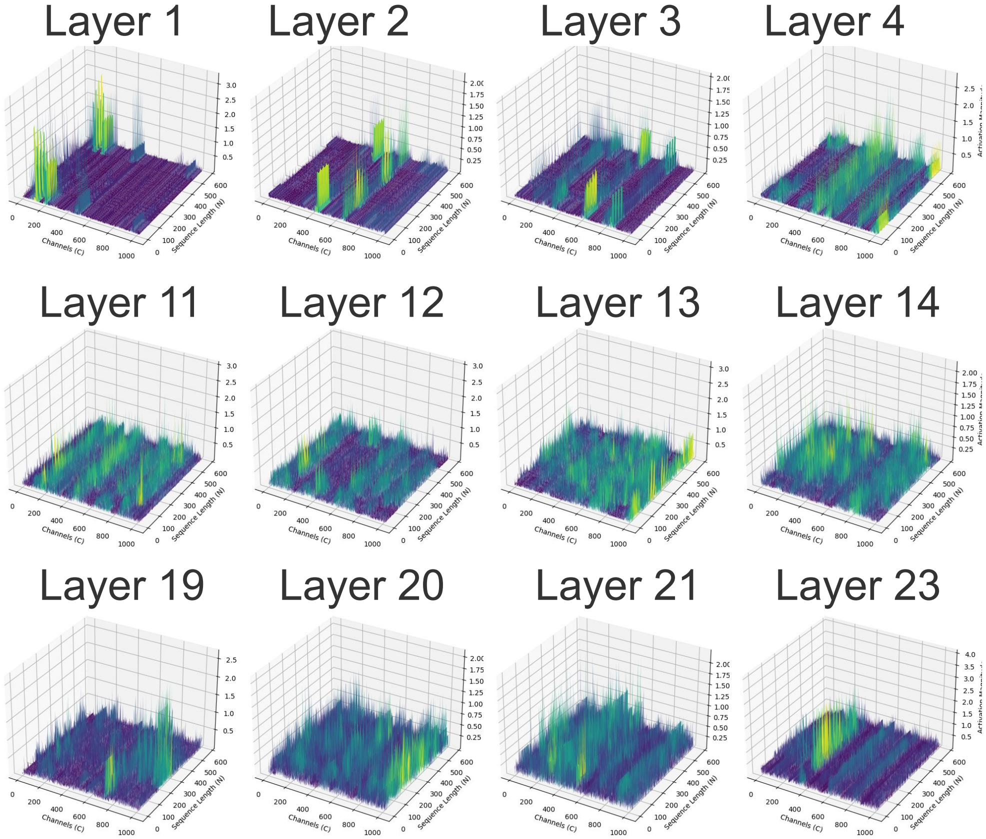
Outlier Study: CLIP

- 3D plots of x8 across layers.



- x8 exhibits channel wise outlier



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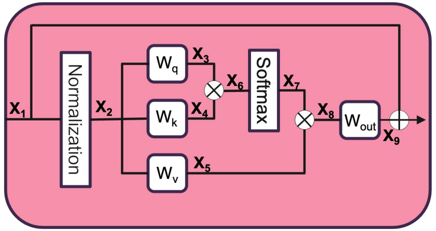
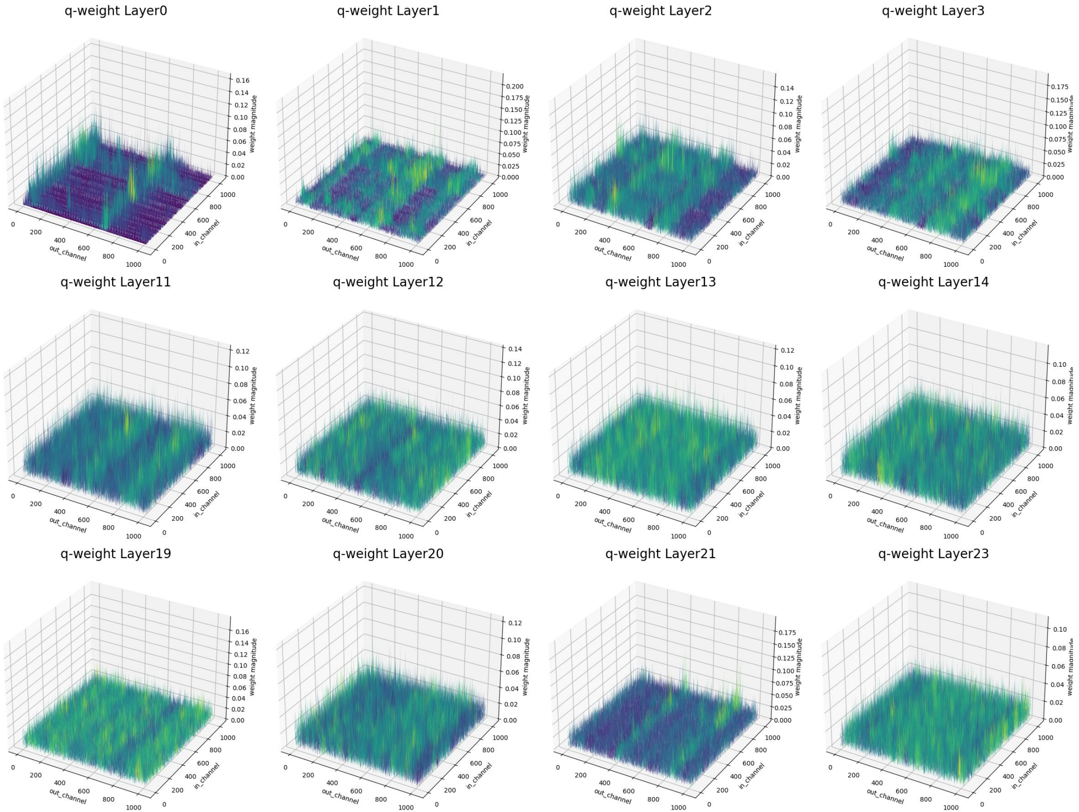
Outlier Study: CLIP Weights

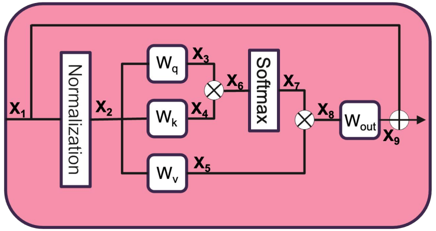
- W_q across CLIP layers.

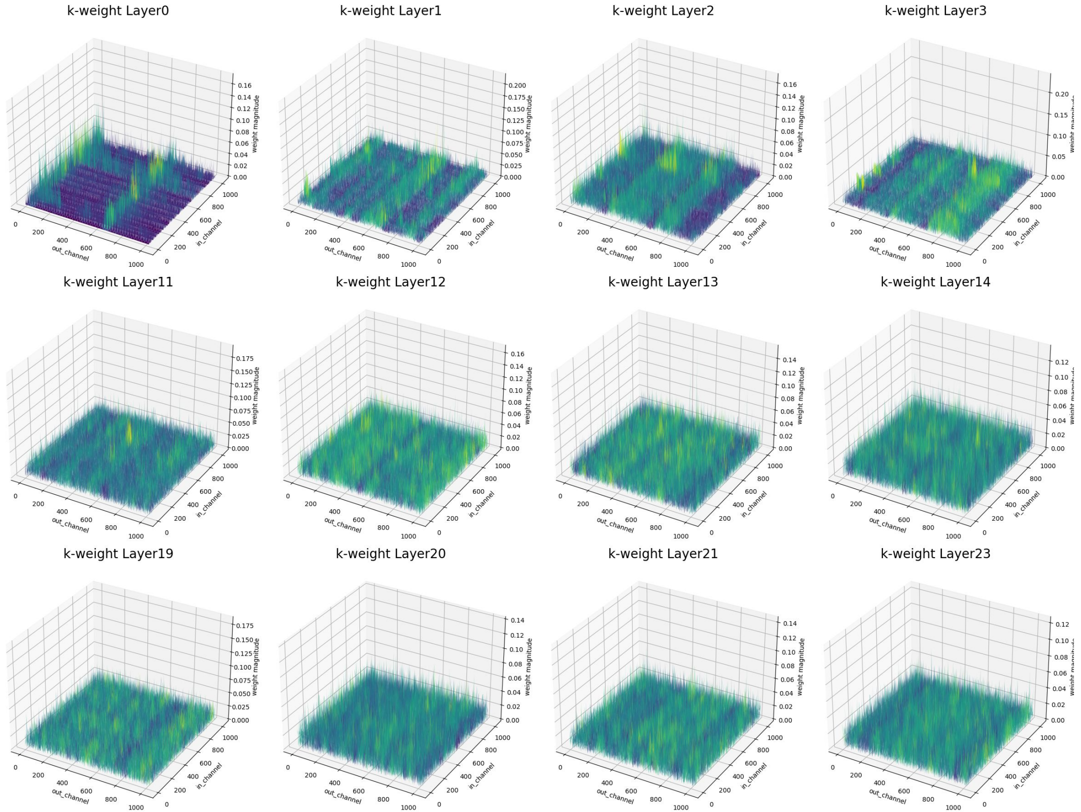




Outlier Study: CLIP Weights

- W_K across CLIP layers.





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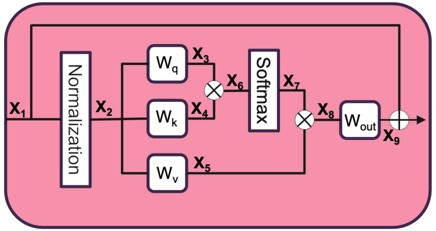
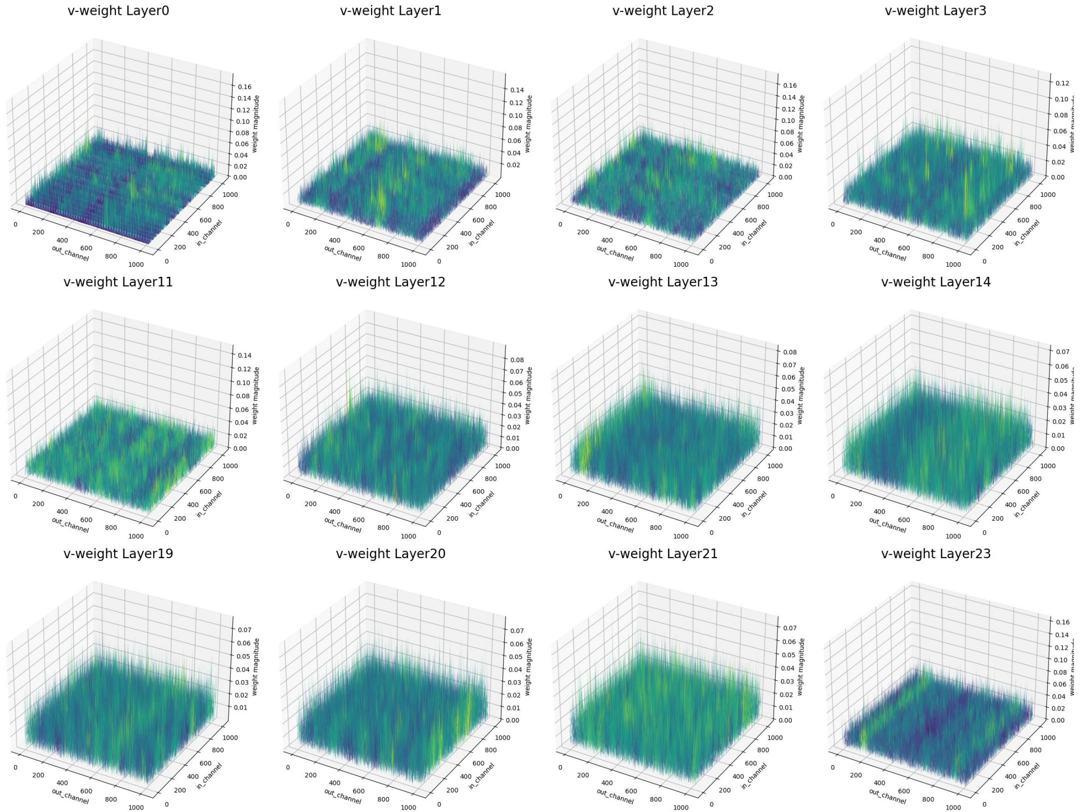
Outlier Study: CLIP Weights

- W_v across CLIP layers.





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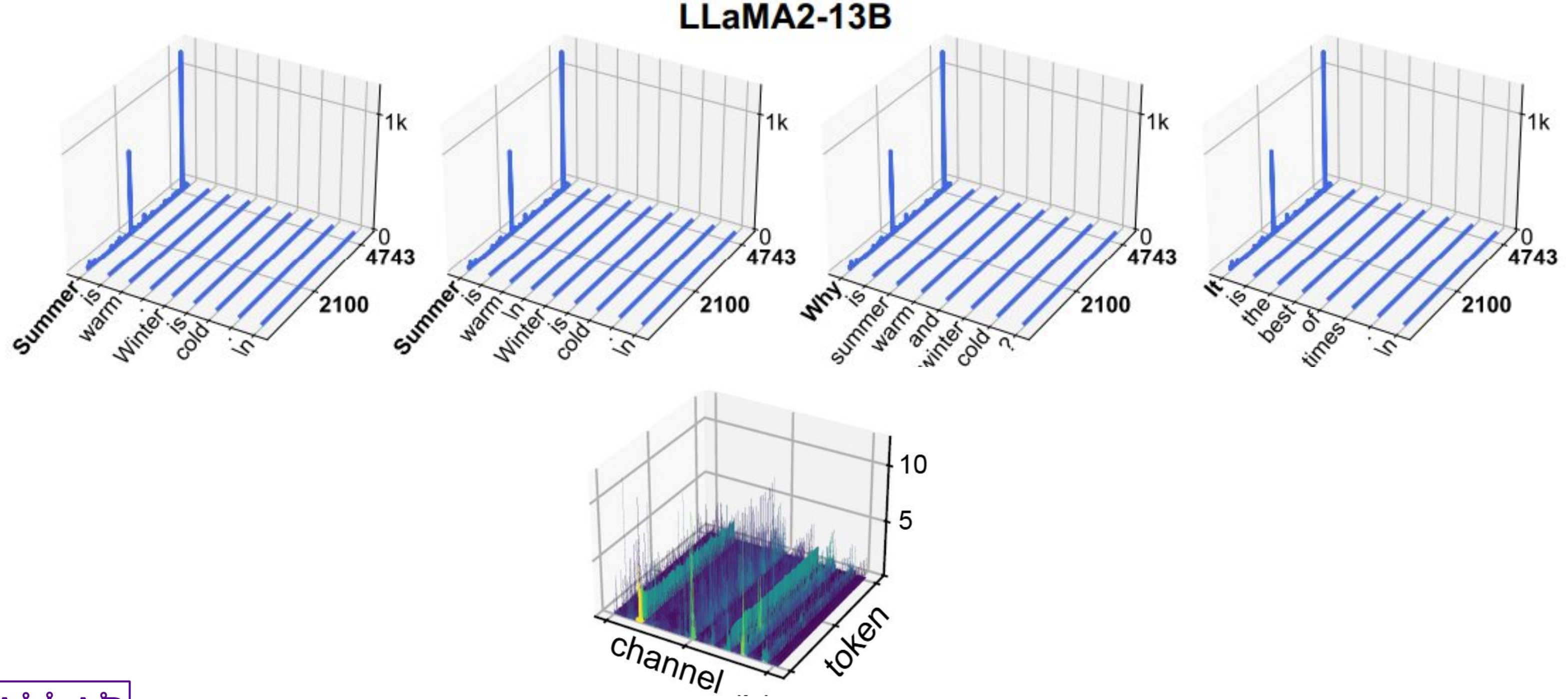
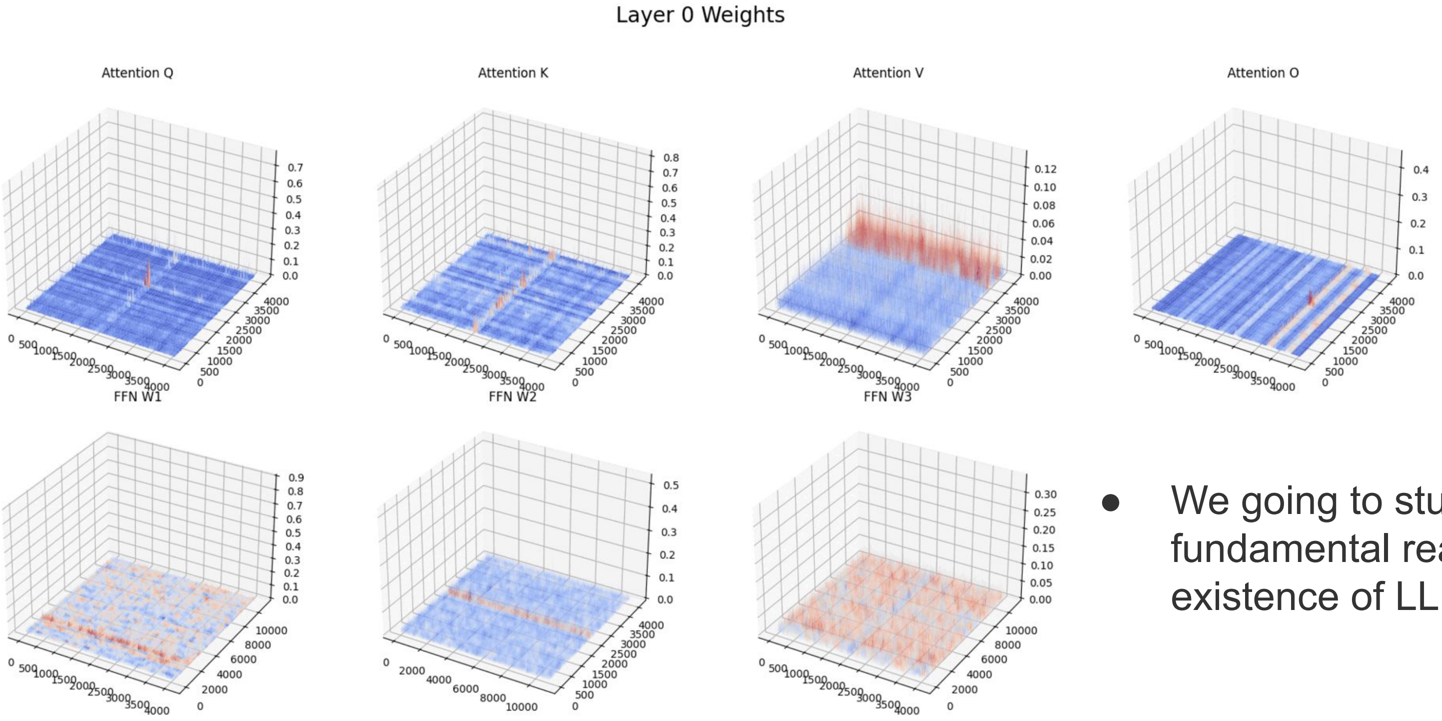
Outlier Study: LLaMA Activations



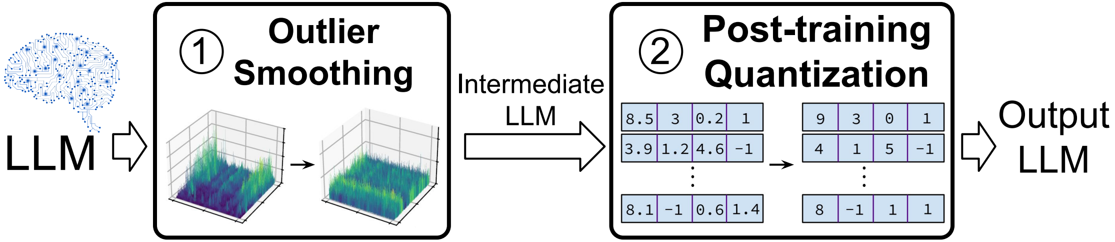
Sun, Mingjie, et al. "Massive activations in large language models." *arXiv preprint arXiv:2402.17762* (2024).

Study the Reason of LLM Outliers



- We going to study the fundamental reason of the existence of LLM outliers.

Outlier Smoothing



- When performing post-training quantization on a LLM, it's common to include a step of outlier smoothing prior to the quantization process.

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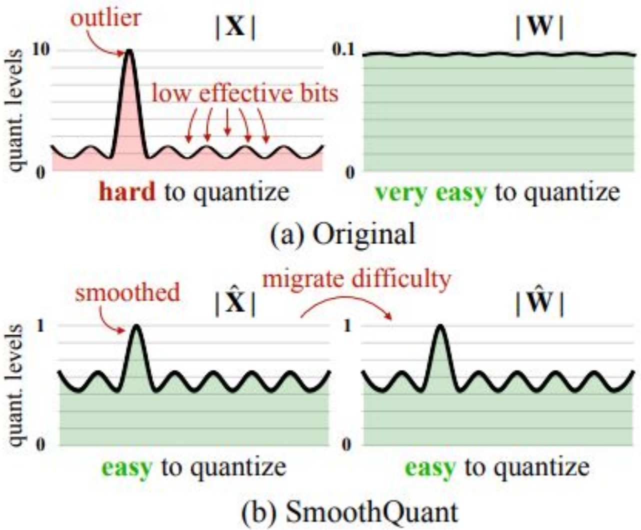
SmoothQuant



- The intermediate results within LLM usually have a lot of outliers.

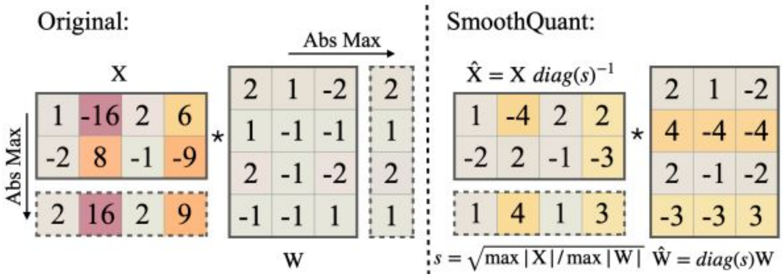
- SmoothQuant smooths the activation outliers by offline migrating the quantization difficulty from activations to weights with a mathematically equivalent transformation.

$$\mathbf{Y} = (\mathbf{X}\mathrm{diag}(\mathbf{s})^{-1}) \cdot (\mathrm{diag}(\mathbf{s})\mathbf{W}) = \hat{\mathbf{X}}\hat{\mathbf{W}}$$

- s depends on the square root of the magnitude of the largest channel

Xiao, Guangxuan, et al. "Smoothquant: Accurate and efficient post-training quantization for large language models." *International Conference on Machine Learning*. PMLR, 2023.

SmoothQuant

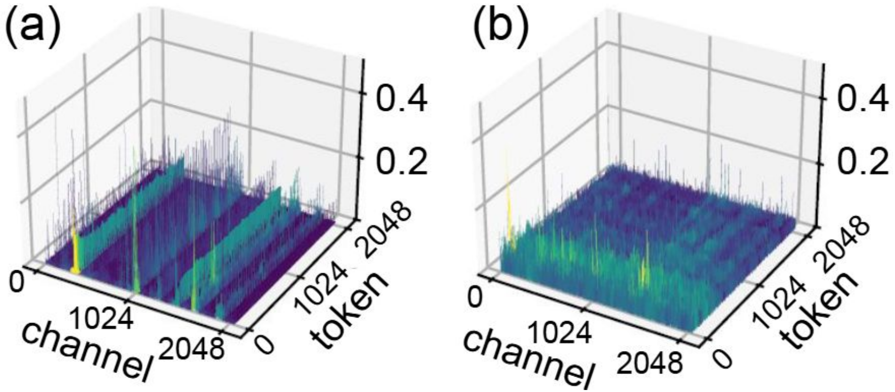


$$\mathbf{Y} = (\mathbf{X}\mathrm{diag}(\mathbf{s})^{-1}) \cdot (\mathrm{diag}(\mathbf{s})\mathbf{W}) = \hat{\mathbf{X}}\hat{\mathbf{W}}$$

$$\mathbf{s}_j = \max(|\mathbf{X}_j|)^{\alpha} / \max(|\mathbf{W}_j|)^{1-\alpha}$$

OPT-175B	LAMBADA	HellaSwag	PIQA	WinoGrande
FP16	74.7%	59.3%	79.7%	72.6%
W8A8	0.0%	25.6%	53.4%	50.3%
ZeroQuant	0.0%*	26.0%	51.7%	49.3%
`LLM.int8()`	74.7%	59.2%	79.7%	72.1%
Outlier Suppression	0.00%	25.8%	52.5%	48.6%
SmoothQuant-O1	74.7%	59.2%	79.7%	71.2%
SmoothQuant-O2	75.0%	59.0%	79.2%	71.2%
SmoothQuant-O3	74.6%	58.9%	79.7%	71.2%

QuaRot



- QuaRot introduces a novel methods to convert the weights and activation of LLM.
- After conversion, most of the outliers within the activation and weights are removed.
- This conversion introduces almost no additional cost during the inference.

NYU SAI LAB Ashkboos, Saleh, et al. "Quarot: Outlier-free 4-bit inference in rotated llms." *arXiv preprint arXiv:2404.00456* (2024).

QuaRot

- Assume Y = AW, where A may have outliers, quantizing A and W as Q(A) and Q(W) could result in increased quantization error. Consequently, Q(A)Q(W) may differ significantly from AW.
- With QuaRot, a orthogonal matrix is applied to eliminate the outliers within A.

$$A \longrightarrow \boxed{W} \longrightarrow AW \qquad\qquad AR \longrightarrow \boxed{R^TW} \longrightarrow AW$$

$$R^TR=RR^T=I$$

$$Q(A) \longrightarrow \boxed{Q(W)} \longrightarrow Q(A)Q(W) \qquad\qquad Q(AR) \longrightarrow \boxed{Q(R^TW)} \longrightarrow Q(AR)Q(R^TW)$$

- R^TW can be computed offline, AR can be generated by modifying the weight matrices of the last layer.

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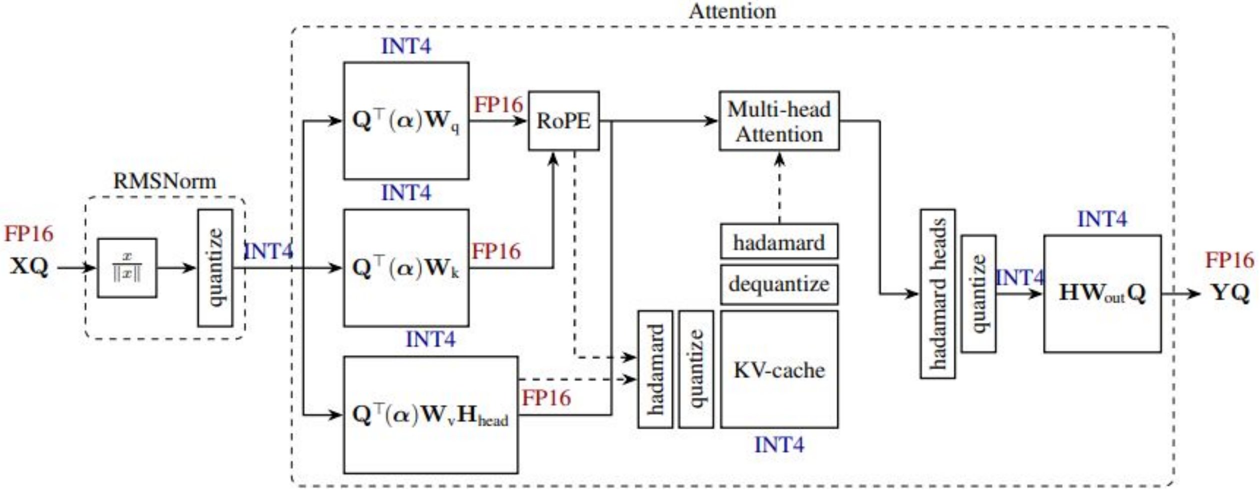
QuaRot



- For some of the layers, the conversion needs to be performed online
- We can use Hadamard matrix, which consists of only 1 and -1 to facilitate the matrix multiplications.

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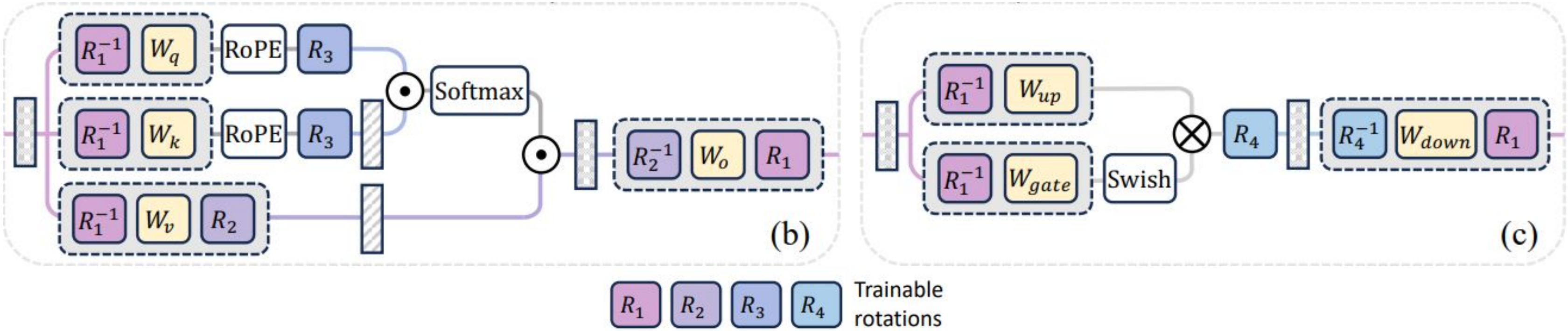
SpinQuant



$$\arg\min_{R \in \mathcal{M}} \mathcal{L}_Q(R_1, R_2 \mid W, X)$$

- SpinQuant optimizes (or learns) the rotation matrices to obtain the minimal changes on the training loss.
- We have to ensure the rotational matrix still satisfies the orthogonal property → Cayley Optimization.

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Liu, Zechun, et al. "SpinQuant--LLM quantization with learned rotations." *arXiv preprint arXiv:2405.16406* (2024).